Re-sit Exam Introduction to Logic (CS & MA)

Friday 2 December, 2016, 18.30 - 21.30 h.

- ☞ Only write your student number at the top of the exam, not your name. Also write your student number at the top of any additional pages. In this way we will be able to grade anonymously.
- ☞ Use a blue or black pen (so no pencils, red pen or marker).
- ☞ Leave the first ten lines of the first page blank (this is where the calculation of your grade will be written).
- ☞ Only hand in your definite answers. You can take any drafts home.
- ☞ When you hand in your exam, wait until the supervisors have checked whether all information is complete. They will indicate when you can leave.
- ☞ By writing your student number on all pages, you earn a first 'free' 10 points. With the regular exercises, you can earn 90 points. With the bonus exercise, you can earn 10 extra points.
  The re-sit exam grade is:
  (the number of points you earned with the regular and bonus exercises + the first 'free' 10) divided by 10, with a maximum grade of 10.

Good luck!

1: Translating to propositional logic (10 points) Translate the following sentences to *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation keys.

a. I will move neither to Utrecht nor to Groningen, unless you persuade me.

b. Ben Feringa will travel to Stockholm precisely if Janine Jansen will play at the Nobel concert.

2: Translating to first-order logic (10 points) Translate the following sentences to *first-order logic*. Do not forget to provide the translation key: only one key for the entire exercise. The domain of discourse is the set of all people.

a. If Carlsen has beaten Karjakin, then there is a grandmaster who is better than all other grandmasters.

b. Even though all Carlsen's Facebook friends have followed the match, not all of them prefer him over Karjakin.

3: Formal proofs (20 points) Give formal proofs of the following inferences. Do not forget the justifications. You can only use the Introduction and Elimination rules and the Reiteration rule.

a.
$$\begin{array}{|l} (A \lor B) \to (C \land D) \\ \hline (A \to C) \land (B \to D) \end{array}$$

b.
$$\begin{array}{|l} R(a,b) \land R(b,a) \\ a = b \\ \hline R(a,a) \end{array}$$

c.
$$\begin{array}{|l} \forall x(P(x) \to Q(x)) \\ \neg\exists z Q(z) \\ \hline \neg\exists y P(y) \end{array}$$

d.
$$\begin{array}{|l} \forall x R(x,x) \\ \hline \forall w \exists z R(z,w) \end{array}$$

4: Truth tables (10 points) Answer the following questions using truth tables. Write down the complete truth tables and motivate your answers.

a. Check with a truth table whether the conclusion is a tautological consequence of the premises in the following argument.

$$\begin{array}{|l} (A \land \neg B) \leftrightarrow (C \land \neg B) \\ \hline (A \leftrightarrow C) \land \neg B \end{array}$$

b. Check with a truth table whether the conclusion is a logical consequence of the premise in the following argument. Indicate clearly which rows are spurious.

$$\begin{array}{|l} \neg\mathsf{Larger}(\mathsf{a},\mathsf{b}) \to \mathsf{Large}(\mathsf{a}) \\ \hline (\mathsf{Larger}(\mathsf{b},\mathsf{a}) \to \neg\mathsf{Large}(\mathsf{a})) \land \neg\mathsf{Large}(\mathsf{a}) \end{array}$$

5: Normal forms propositional logic (5 points)

Provide a conjunctive normal form (CNF) of the following formula. Show all of the intermediate steps.

$$\neg((P \to Q) \land ((Q \land R) \lor \neg(S \lor \neg R)))$$

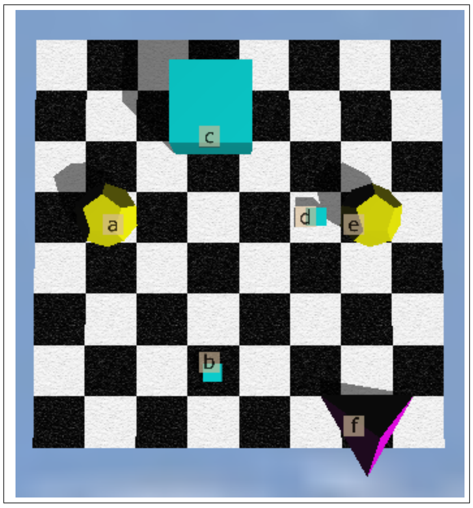
6: Translating function symbols (5 points)

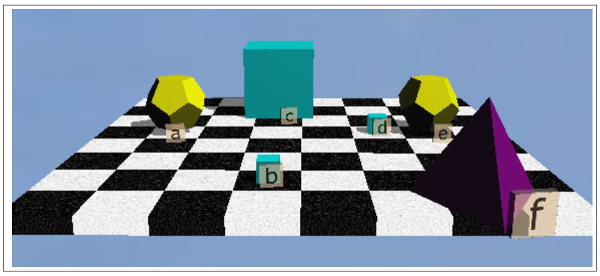
Translate the following sentences to first-order logic. Provide only one translation key for the entire exercise. The translation key should at least contain the function *succ* where $succ(x)$ corresponds to: "the successor of x". The domain of discourse is the set of all presidents of the United States of America.

a. No one is the husband of anyone's successor.

b. George H.W. Bush is the father of the successor of the successor of his successor.

7: Tarski's World (10 points) In the world displayed below, b and d are small, c and f are large and the other objects are medium.





a. In the world displayed above, there is at least one small cube which is in the same row of at least two dodecahedrons. How can you express this with one formula in the language of Tarski's World such that the formula would be true in every world with at least one small cube that is in the same row of at least two dodecahedrons, and false otherwise?

b. Indicate of each formula below, whether it is true or false in the world displayed above. You do not need to explain your answers.

(i) $(\mathsf{Small}(\mathsf{c}) \vee \mathsf{SameRow}(\mathsf{b},\mathsf{f})) \leftrightarrow \mathsf{Between}(\mathsf{d},\mathsf{a},\mathsf{e})$

(ii) $\mathsf{Smaller}(\mathsf{e},\mathsf{f}) \rightarrow ((\mathsf{BackOf}(\mathsf{d},\mathsf{e}) \rightarrow \mathsf{SameSize}(\mathsf{b},\mathsf{d})) \wedge \neg\mathsf{Cube}(\mathsf{d}))$

(iii) $\exists \mathsf{x}(\mathsf{Large}(\mathsf{x}) \wedge \neg\mathsf{Dodec}(\mathsf{x}) \wedge \mathsf{RightOf}(\mathsf{x},\mathsf{a})) \rightarrow \forall \mathsf{x}\forall \mathsf{y}\ (\mathsf{x} \neq \mathsf{y} \rightarrow \mathsf{SameRow}(\mathsf{x},\mathsf{y}))$

(iv) $\neg\forall \mathsf{x}(\mathsf{LeftOf}(\mathsf{x},\mathsf{f}) \rightarrow \mathsf{Cube}(\mathsf{x}))$

c. Explain how the formula below can be made true by removing one object from the world displayed above.

$$\forall \mathsf{x}((\neg\mathsf{Cube}(\mathsf{x}) \rightarrow (\exists \mathsf{y}\mathsf{BackOf}(\mathsf{y},\mathsf{x}) \wedge \mathsf{Cube}(\mathsf{y}))) \vee \mathsf{Cube}(\mathsf{x}))$$

8: Normal forms first-order logic (10 points)

a. Provide a Prenex normal form of the following formula. Show all of the intermediate steps.
$\forall x P(x) \leftrightarrow Q(c)$

b. Provide a Skolem normal form of the following formula. Show all of the intermediate steps.
$\exists x \forall y \forall z \exists u (R(x,y,z) \vee P(u))$

c. Check the satisfiability of the Horn sentence below using the Horn algorithm. If you prefer the conditional form, you may also use the satisfiability algorithm for conditional Horn sentences.

$$(\neg A \vee \neg B \vee \neg C) \wedge (B \vee \neg D) \wedge C \wedge (\neg E \vee A) \wedge (E \vee \neg C)$$

9: Semantics (10 points)

Let a model $\mathfrak{M}$ with domain $\mathfrak{M}(\forall) = \{1, 2, 3, 4\}$ be given. There are four constants: a, b, c, d. There are three predicate symbols: R and P. We have:

- $\mathfrak{M}(a) = 2$, $\mathfrak{M}(b) = 3$, $\mathfrak{M}(c) = 4$, $\mathfrak{M}(d) = 4$;
- $\mathfrak{M}(R) = \{\langle 1, 1\rangle, \langle 2, 2\rangle, \langle 2, 3\rangle, \langle 3, 3\rangle\}$;
- $\mathfrak{M}(P) = \{3, 4\}$.

Let h be an assignment such that: $h(x) = 1$, $h(y) = 2$, and $h(z) = 3$.
Evaluate the following statements. Follow the truth definition step by step.

a. $\mathfrak{M} \models P(x) \wedge R(x, z)[h[x/4]]$

b. $\mathfrak{M} \models \forall x \exists y \neg R(x, y)[h]$

c. $\mathfrak{M} \models \exists y \forall x (P(y) \rightarrow (R(y, x) \vee R(x, z)))[h]$

10: Bonus question (10 points)

Suppose it's the day of the midterm and four students, namely Alex (guy), Bobby (guy or girl), Chris (guy or girl) and Dave (guy), are taking part. The 4 students have different fields of study; they have earned different numbers of ECs in their studies, all of them multiples of 5, between 5 and 40 EC inclusive; and all four hand in the midterm at different times.
Can you find out for all four students what is their field of study, number of ECs, and time of handing in the midterm?
Explain in a sentence each why your solution makes all 9 sentences below true.
The combination of the following cues should suffice. All resemblances to real life are purely accidental.

a. Alex had a beer together with two other students, namely the Law student and the student who handed in the midterm at 10:30.

b. One of the students that do not study Mathematics has precisely 3 times as many ECs as the Mathematics student, UNLESS David studies Computing Science.

c. Bobby did NOT submit the midterm at 10:00.

d. Alex has more ECs than David.

e. The Computing Science student submitted the midterm earlier than the student who has exactly $\frac{1}{5}$ of the EC of the Computing Science student, BUT later than Chris.

f. NEITHER the Mathematics student NOR the woman who has 15 EC handed in the midterm at 10:00.

g. The student who handed in the midterm at 9:30 has exactly twice as many ECs as the student of Astronomy.

h. The student who submitted the midterm at 11:00 is a man IF AND ONLY IF Bobby studies Law.

i. The Mathematics student was not the last one to hand in the midterm.